NONUNIQUENESS OF AUTOWAVE REGIMES OF PROPAGATION OF SWITCHING WAVES

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It is shown theoretically that the relaxation of the heat flux for certain values of the parameters of the system results in nonuniqueness of autowave regimes of switching waves.

The transitions superconductor-normal metal [1], semiconductor—semimetal [2], and diffusion-kinetic regime in heterogeneous catalysis [3] are waves of switching of uniform states in bistable systems. The propagation of such waves is usually described by the equation of heat conduction of a parabolic type [1-4], following from Fourier's law and the law of conservation of energy. However at low temperatures in solids [5] or when the velocities of propagation of the waves is high [1] Fourier's law must be replaced by a relation that takes into account the relaxation of the heat flux q [5, 6]:

$$\tau \frac{\partial q}{\partial t} + q = -\lambda \operatorname{grad} T. \tag{1}$$

This work is devoted to studying the effect of the relaxation of the heat flux (1) on the autowave regimes of propagation of low-temperature waves of switching. The law of conservation of energy and Eq. (1) give an equation for determining the temperature. In the coordinate system tied to the wave this equation has the following form [7]:

$$(1-\varphi^2)\frac{d^2\theta}{dx^2} - \varphi(1+B)\frac{d\theta}{dx} + \omega(x) - B\theta + \varphi\frac{d\omega}{dx} = 0, \tag{2}$$

where $\varphi = V/v$; $v = (a/\tau)^{1/2}$ [6]; $\theta = (T-T_0)(T_m-T_0)^{-1}$, where T_0 and T_m are the temperatures of the uniform states of the system; $\omega(x)$ is the dimensionless intensity of the heat source; $B = \alpha\tau/(c\rho H)$; and H is the thickness of the layer in which the wave propagates. We note that Eq. (2) differs significantly from the parabolic heat-conduction equation: the coefficient multiplying the second derivative depends on the dimensionless velocity of the wave φ, and the coefficient mutliplying the first derivative depends on B. In addition, Eq. (2) contains a derivative of the source with respect to the coordinate.

We shall study a piecewise-linear heating function ω, which grows linearly with the temperature in a wavefront of width δ while outside it the function is constant (for $\delta = 0$ such a dependence acquires the form of a "step" [1, 7]). Then from Eq. (2) we obtain a relation for determining the dimensionless velocity of the wave φ :

$$\theta^* = \frac{B(1+\gamma_1\varphi)\delta}{(1-\varphi^2)(\gamma_1-\gamma_2)(\exp\gamma_1\delta - 1)} \equiv f(\varphi), \tag{3}$$

where

$$\gamma_{1,2} = \frac{1}{2}\frac{\varphi(1+B)}{1-\varphi^2} \pm \frac{1}{2}\left(\frac{\varphi^2(1+B)^2}{(1-\varphi^2)^2} + \frac{4B}{1-\varphi^2}\right)^{1/2};$$

and θ^* is the dimensionless temperature of the uniform stable state of the system ($0 < \theta^* < 1$).

Figure 1 shows graphs of the right side of Eq. (3) for different values of the parameter B. The abscissas of the points of intersection of $f(\varphi)$ and $\theta^* = \text{const}$ are equal to the

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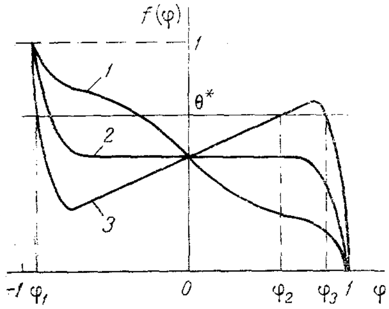

Fig. 1. The dependence of the temperature in the wavefront $f(\varphi)$ for different values of the parameter B: 1) $B < B^*$; 2) $B = B^*$; 3) $B > B^*$.

velocities of the wave φ in the autowave regime of propagation. As $\theta^* \to 0$ $\varphi \to 1$, i.e., the velocity of the wave of switching V approaches the velocity of propagation of heat in the medium v. Autowave regimes with $|V| > v$ do not exist (phase waves can propagate with velocities $|V| > v$). If $B < B^* = 1$, the solution is unique for any values of $0 < \theta^* < 1$. For $B > B^*$ the curve $f(\varphi)$ has a local minimum f_{min} and maximum f_{max}. For this reason for $f_{min} < \theta^* < f_{max}$ there exist three autowave regimes of propagation of switching waves (the question of the stability of these regimes requires a special analysis). The nonuniqueness of the solution is caused by relaxation, namely, by the effect of the derivative of the source function in the heat-conduction equation (the last term in Eq. (2)), which for $\varphi<0$ results in a reduction of the effective heating in the wavefront and for $\varphi>0$ in an increase of the effective heating. As a result of this for $B > B^* = 1$ there appear maxima and minima in the dependence $f(\varphi)$.

Thus the relaxation of the heat flux strongly affects the propagation of waves of the transition superconductor—normal metal. The generality of the mathematical model makes it possible to transfer the results obtained to other types of low-temperature switching waves also.

NOTATION

q, heat flux; τ, relaxation time; t, time; λ, thermal conductivity; T, temperature; V, velocity of propagation of the wave; v, velocity of propagation of heat; θ, dimensionless temperature; B, dimensionless criterion, the ratio of the relaxation time to the characteristic heat transfer time into the surrounding medium; φ, dimensionless velocity of the wave; ω, dimensionless heat source; a, thermal diffusivity; δ, width of the wavefront; θ*, dimensionless temperature of the unstable stationary state; and α, coefficient of heat transfer.

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